Limit at a Point

1. If $\lim_{x \to a} f(x) = L$, which of the following must be true?

   (a) $f(a) = L$

   (b) $f(x) = L$

   (c) $\lim_{x \to a^-} f(x) = L$

   (d) $\lim_{x \to a^+} f(x) = L$

   (e) $f(x) \neq L$ for all $x \neq a$.

2. Consider the function f defined by $f(x) = \begin{cases} 3x^2 - 4, & x < 1 \\ 2, & x = 1 \\ 6x - 7, & x > 1 \end{cases}$.

   Which of the following are true statements about this function?

   (Select ALL that are correct.)

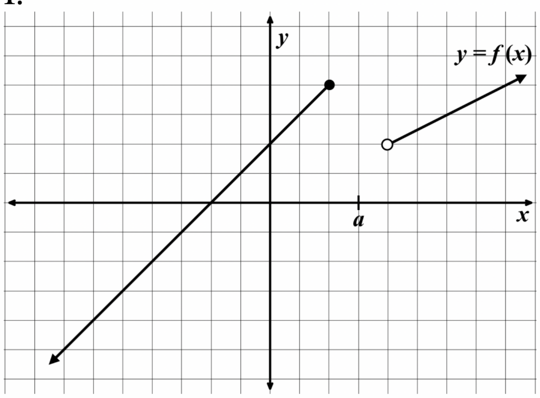
   (a) $\lim_{x \to 1} f(x)$ exists.

   (b) $f(1)$ exists

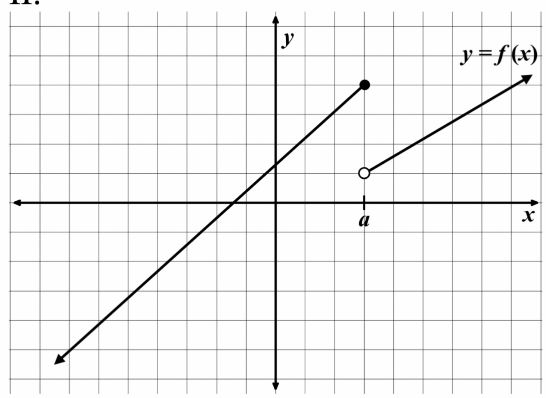
   (c) $\lim_{x \to 1} f(x) = f(1)$

3. For which of the following graphs does $\lim_{x \to a} f(x)$ exist?

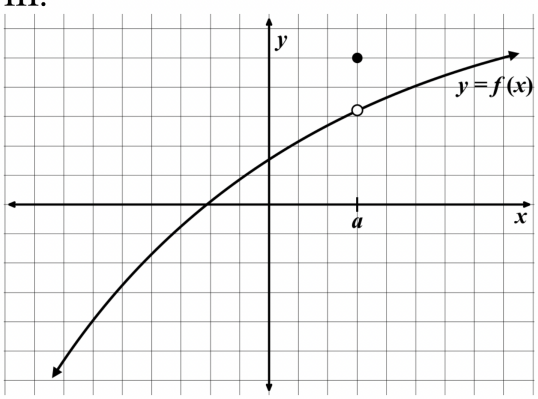
I.



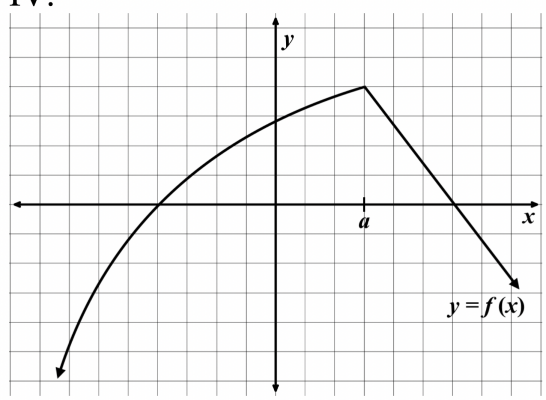
II.



III.



IV.



a. IV only

b. III and IV only

c. II, III, and IV only

d. I, II, III, and IV

e. None of these

4. Suppose that a function $y = f(x)$ has a jump discontinuity at an input value $x = a$. Which of the following statements must be true if the function f is continuous at all other input values?

   a. $f(a)$ must be undefined
   b. $f(a)$ must be defined
   c. $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist and are not equal
   d. $\lim_{x\to a^-} f(x) = f(a)$ or $\lim_{x\to a^+} f(x) = f(a)$
   e. $\lim_{x\to a^-} f(x) = f(a)$ and $\lim_{x\to a^+} f(x) = f(a)$

5. The functions f, g, and h are defined as follows:

$$f(x) = \frac{x^2 - 1}{x - 1} \qquad g(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x \neq 1 \\ 1, & x = 1 \end{cases} \qquad h(x) = x + 1$$

   Which of the following is true?

   I. $\lim_{x\to 1} g(x) = g(1)$
   II. $\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x) = \lim_{x\to 1} h(x)$
   III. $f(1) = g(1) = h(1)$

   a. I only
   b. I and II
   c. II only
   d. II and III
   e. I, II, and III